

Simplify each expression by writing it in terms of the sine or cosine of one angle.

1. $\sin 30^\circ \cos 45^\circ + \cos 30^\circ \sin 45^\circ$

2. $\cos 30^\circ \cos 45^\circ - \sin 30^\circ \sin 45^\circ$

3. $\sin \frac{\pi}{4} \cos \frac{2\pi}{3} - \cos \frac{\pi}{4} \sin \frac{2\pi}{3}$

4. $\cos \frac{7\pi}{6} \cos \frac{\pi}{3} + \sin \frac{7\pi}{6} \sin \frac{\pi}{3}$

Use the sum and difference identities for sine, cosine, and tangent to find the exact value of each trigonometric function.

5. $\cos 75^\circ$

6. $\cos 195^\circ$

7. $\cos\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$

8. $\cos\left(\frac{7\pi}{6} + \frac{\pi}{4}\right)$

9. $\sin 75^\circ$

10. $\sin 195^\circ$

11. $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

12. $\sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right)$

13. $\tan 255^\circ$

14. $\tan 105^\circ$

15. $\tan\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$

16. $\tan\left(\frac{4\pi}{3} + \frac{3\pi}{4}\right)$

True or False.

17. $\cos 2(35^\circ) = 1 - 2 \sin^2 35^\circ$

18. $\cos 2(30^\circ) = 2 \cos^2 60^\circ - 1$

19. $\sin(-40^\circ) = 2 \sin(-20^\circ) \cos(-20^\circ)$

20. $\sin 2(36^\circ) = 2 \sin 72^\circ \cos 72^\circ$

21. $\tan 2(35^\circ) = \frac{2 \tan 70^\circ}{1 - \tan^2 35^\circ}$

22. $\tan(-70^\circ) = \frac{2 \tan(-35^\circ)}{1 - \tan^2(-35^\circ)}$

Use double-angle identities to find the exact value of each trigonometric function.

23. Find $\sin 2\alpha$, if $\cos \alpha = \frac{4}{5}$ and $\frac{3\pi}{2} < \alpha < 2\pi$

24. Find $\cos 2\theta$, if $\sin \theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$

25. Find $\tan 2u$, if $\cos u = \frac{5}{13}$ and $\frac{3\pi}{2} < u < 2\pi$

Use half-angle identities to find exact value of each function. Assume $0 < \theta < 2\pi$

26. Find $\cos \frac{\theta}{2}$, if $\cos \theta = \frac{4}{5}$ and θ lies in quadrant I

27. Find $\tan \frac{\alpha}{2}$, if $\tan \alpha = -2$ and α lies in quadrant II

28. Find $\sin \frac{u}{2}$, if $\cos u = \frac{\sqrt{2}}{2}$ and u lies in quadrant I