PreCalculus Name Key

Sum-Difference & Multiple Angles Formulas Date _____ Period _____

Simplify each expression by writing it in terms of the sine or cosine of one angle.

1. $\sin 30° \cos 45° + \cos 30° \sin 45°$

$\sin 75°$

2. $\cos 30° \cos 45° - \sin 30° \sin 45°$

$\cos 75°$

3. $\sin\frac{\pi}{4}\cos\frac{2\pi}{3} - \cos\frac{\pi}{4}\sin\frac{2\pi}{3}$ $\frac{3}{12} - \frac{8}{12}$

$\sin\left(-\frac{5\pi}{12}\right)$

4. $\cos\frac{7\pi}{6}\cos\frac{\pi}{3} + \sin\frac{7\pi}{6}\sin\frac{\pi}{3}$ $\frac{7}{6} - \frac{2}{6}$

$\cos\left(\frac{5\pi}{6}\right)$

Use the sum and difference identities for sine, cosine, and tangent to find the exact value of each trigonometric function.

5. $\cos 75°$

$\cos 120° \cos 45° + \sin 120° \sin 45°$

$\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

6. $\cos 195°$

$\cos 240° \cos 45° + \sin 240° \sin 45°$

$\left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{-(\sqrt{2}+\sqrt{6})}{4}$

7. $\cos\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$

$\cos\frac{5\pi}{3}\cos\frac{\pi}{4} - \sin\frac{5\pi}{3}\sin\frac{\pi}{4}$

$\left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{-\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}+\sqrt{6}}{4}$

8. $\cos\left(\frac{7\pi}{6} + \frac{\pi}{4}\right)$

$\cos\frac{7\pi}{6}\cos\frac{\pi}{4} - \sin\frac{7\pi}{6}\sin\frac{\pi}{4}$

$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

9. $\sin 75°$

$\sin 120° \cos 45° - \cos 120° \sin 45°$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

10. $\sin 195°$

$\sin 240° \cos 45° - \cos 240° \sin 45°$

$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(-\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

11. $\sin\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$\sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$

$\frac{\sqrt{6}+\sqrt{2}}{4}$

12. $\sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right)$

$\sin\frac{\pi}{4}\cos\frac{4\pi}{3} + \cos\frac{\pi}{4}\sin\frac{4\pi}{3}$

$\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right)$

$\frac{-(\sqrt{2}+\sqrt{6})}{4}$

13. $\tan 255°$

$\frac{\tan 210° + \tan 45°}{1 - \tan 210° \tan 45°} = \frac{\frac{\sqrt{3}+3}{3}}{\frac{3-\sqrt{3}}{3}} = 2+\sqrt{3}$

14. $\tan 105°$

$\frac{\tan 60° + \tan 45°}{1 - \tan 60° \tan 45°} = \frac{\sqrt{3}+1}{1-\sqrt{3}} = -2-\sqrt{3}$

15. $\tan\left(\frac{5\pi}{3} + \frac{\pi}{4}\right)$

$\frac{\tan\frac{5\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{5\pi}{3}\tan\frac{\pi}{4}} = \frac{-\sqrt{3}+1}{1+\sqrt{3}} \Rightarrow \sqrt{3}-2$

16. $\tan\left(\frac{4\pi}{3} + \frac{3\pi}{4}\right)$

$\frac{\tan\frac{4\pi}{3} + \tan\frac{3\pi}{4}}{1 - \tan\frac{4\pi}{3}\tan\frac{3\pi}{4}} = \frac{\sqrt{3}-1}{1+\sqrt{3}} = 2-\sqrt{3}$

Jean Adams

True or False.

17. $\cos 2(35°) = 1 - 2\,sin^2\ 35°$

True

18. $\cos 2(30°) = 2\,cos^2\ 60° - 1$

False

19. $\sin(-40°) = 2\sin(-20°)\cos(-20°)$

True

20. $\sin 2(36°) = 2\sin 72°\cos 72°$

False

21. $\tan 2(35°) = \frac{2\tan 70°}{1 - tan^2\ 35°}$

False

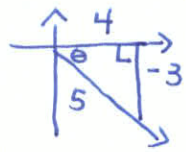
22. $\tan(-70°) = \frac{2\tan(-35°)}{1 - tan^2(-35°)}$

True

Use double-angle identities to find the exact value of each trigonometric function.

23. Find $\sin 2\alpha$, if $\cos\alpha = \frac{4}{5}$ and $\frac{3\pi}{2} < \alpha < 2\pi$

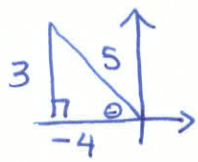
$2\sin\alpha\cos\alpha$



$2\left(-\frac{3}{5}\right)\left(\frac{4}{5}\right) = -\frac{24}{25}$

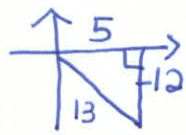
24. Find $\cos 2\theta$, if $\sin\theta = \frac{3}{5}$ and $\frac{\pi}{2} < \theta < \pi$

$1 - 2\sin^2\theta$



$1 - 2\left(\frac{3}{5}\right)^2 = \frac{25}{25} - \frac{18}{25} = \frac{7}{25}$

25. Find $\tan 2u$, if $\cos u = \frac{5}{13}$ and $\frac{3\pi}{2} < u < 2\pi$



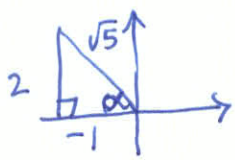
$\frac{2\left(\frac{12}{5}\right)}{1 - \left(\frac{12}{5}\right)^2}\ \ \frac{\frac{24}{5}}{\frac{119}{25}} = \frac{24\cdot 5}{119} \Rightarrow \frac{120}{119}$

Use half-angle identities to find exact value of each function. Assume $0 < \theta < 2\pi$

26. Find $cos\frac{\theta}{2}$, if $\cos\theta = \frac{4}{5}$ and θ lies in quadrant I

$\sqrt{\frac{1 + \frac{4}{5}}{2}} = \sqrt{\frac{9}{5}\cdot\frac{1}{2}} \Rightarrow \sqrt{\frac{9}{10}} \Rightarrow \frac{3\sqrt{10}}{10}$

27. Find $tan\frac{\alpha}{2}$, if $\tan\alpha = -2$ and α lies in quadrant II



$\frac{1 + \frac{1}{\sqrt{5}}}{\frac{2}{\sqrt{5}}} \Rightarrow \frac{\frac{\sqrt{5}+1}{\sqrt{5}}}{\frac{2}{\sqrt{5}}} \Rightarrow \frac{\sqrt{5}+1}{2}$

28. Find $sin\frac{u}{2}$, if $\cos u = \frac{\sqrt{2}}{2}$ and u lies in quadrant I

$\sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2}} = \sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}} \Rightarrow \sqrt{\frac{2-\sqrt{2}}{4}} \Rightarrow \frac{\sqrt{2-\sqrt{2}}}{2}$

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