A ***matrix*** is a rectangular array of real numbers.

Matrix *A* has 2 horizontal rows and 3 vertical columns.

Each entry can be identified by its position in the matrix.

$$A = \begin{bmatrix} 3 & 1 & -2 \\ 7 & -1 & 0.5 \end{bmatrix}$$

Order of a Matrix

A matrix with *m* rows and *n* columns is of order *m* x *n*.

If *m* = *n* the matrix is said to be square of order *n*.

7 is in Row 2 Column 1
-2 is in Row 1 Column 3
***A* is of order 2 x 3**

Example 1: Find the order of each matrix.

a.
$$A = \begin{bmatrix} 2 & 3 & 1 & 0 \\ 4 & 2 & 1 & 4 \\ 1 & 1 & 6 & 2 \end{bmatrix}$$

b.
$$B = \begin{bmatrix} 2 & 5 & 2 & -1 & 0 \end{bmatrix}$$

c.
$$C = \begin{bmatrix} 3 & 1 \\ 6 & 2 \end{bmatrix}$$

An *m* x *n* matrix can be written

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

Two matrices $A = [a_{ij}]$ and $B = [b_{ij}]$ are ***equal*** if they have the same order and $a_{ij} = b_{ij}$ for every *i* and *j*.

For example,

$$\begin{bmatrix} 0.5 & \sqrt{9} \\ \frac{1}{4} & 7 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 3 \\ 0.25 & 7 \end{bmatrix}$$

both matrices are of order 2 x 2 and all corresponding entries are equal.

Example 2: Find the sums *A* + *B* and *B* + *C*.

$$A = \begin{bmatrix} 1 & 5 \\ 2 & 1 \\ 0 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 2 & 0 & 6 \\ -1 & 0 & -3 \end{bmatrix} \quad C = \begin{bmatrix} 3 & -3 & 0 \\ 3 & 2 & 4 \end{bmatrix}$$

To ***ADD*** matrices:

1. Check to see if the matrices have the same order.
2. Add corresponding entries.

Example 3: Find the differences $A - B$ and $B - C$.

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 2 & -1 \\ 4 & -5 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 5 & 1 \\ 2 & 1 & 6 \end{bmatrix}$$

To ***SUBTRACT*** matrices:

1. Check to see if the matrices have the same order.
2. Subtract corresponding entries.

Scalar Multiplication

If $A = [a_{ij}]$ is an m x n matrix and c is a scalar (a real number), then the m x n matrix $cA = [ca_{ij}]$ is the ***scalar multiple*** of A by c.

Example 4: Calculate the value of $3A - 2B + C$ with $A = \begin{bmatrix} 2 & -1 \\ 3 & 5 \\ 4 & -2 \end{bmatrix}$ $B = \begin{bmatrix} 5 & 2 \\ 1 & 0 \\ 3 & -1 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & 2 \\ 1 & 0 \\ 3 & -1 \end{bmatrix}$

Example 5: **Using the Distributive Property**

A) $2\left(\begin{bmatrix} 1 & 3 \\ -2 & 2 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ -3 & 1 \end{bmatrix}\right) =$

B) $4\left(\begin{bmatrix} -4 & 0 & 1 \\ 0 & 2 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 1 & -2 \\ 3 & -6 & 0 \end{bmatrix}\right) =$

Properties of Matrix Addition and Scalar Multiplication

Let *A*, *B*, and *C* be *m x n* matrices and let *c* and *d* be scalars.

1. $A + B = B + A$	**Commutative Property of Matrix Addition**
2. $A + (B + C) = (A + B) + C$	**Associative Property of Matrix Addition**
3. $(cd)A = c(dA)$	**Associative Property of Scalar Multiplication**
4. $1A = A$	**Scalar Identity Property**
5. $c(A + B) = cA + cB$	**Distributive Property**
6. $(c + d)A = cA + dA$	**Distributive Property**

Homework: page 579 #9-14 and pages 594-595 #7-23 odd