Precalculus Notes **Lesson 8.1 Matrices and Systems of Equations Part 1**

An ***augmented matrix*** and a ***coefficient matrix*** are associated with systems of linear equations.

For the system

$$\begin{cases} 2x + 3y - z = 12 \\ x - 8y = 16 \end{cases}$$

The ***augmented matrix*** is

$$\left[\begin{array}{ccc} 2 & 3 & -1 \\ 1 & -8 & 0 \end{array} \vdots \begin{array}{c} 12 \\ 16 \end{array}\right]$$

The ***coefficient matrix*** is

$$\begin{bmatrix} 2 & 3 & -1 \\ 1 & -8 & 0 \end{bmatrix}$$

Elementary Row Operations

1. **Interchange any two rows of a matrix.**
2. **Multiply one row of a matrix by a nonzero constant.**
3. **Add a multiple of one row of a matrix another.**

Example 1: Apply the elementary row operation to the augmented matrix of the system.

A) Row Operation — System — Augmented Matrix

$R_1 \leftrightarrow R_2$

$$\begin{cases} x + 2y = 8 \\ 3x - y = 10 \end{cases}$$

B) Row Operation — System — Augmented Matrix

$3R_2$

$$\begin{cases} x + 2y = 8 \\ 3x - y = 10 \end{cases}$$

C) Row Operation — System — Augmented Matrix

$-3R_1 + R_2$

$$\begin{cases} x + 2y = 8 \\ 3x - y = 10 \end{cases}$$

A matrix in *Row-Echelon* form has the following properties.

1. **Any rows consisting entirely of zeros is at the bottom of the matrix.**
2. **The first nonzero entry is 1 for each row that does not entirely contain zeros.**
3. **For two successive (nonzero) rows, the leading 1 in the higher row is farther to the left than the leading one in the lower row.**

A matrix is in *Reduced Row-Echelon* form if every column that has a leading 1 has zeros in every position above and below its leading 1.

Example 2: Determine whether each matrix is in row-echelon form. If it is, determine whether it is in reduced row-echelon form. If it is not, then explain why it is not.

a. $\begin{bmatrix} 1 & 0 & -2 & 4 \\ 0 & 1 & 11 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

b. $\begin{bmatrix} 1 & 3 & 4 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 6 & -1 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 2 & -1 & 8 \\ 0 & 3 & 1 & -4 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 2 & -5 & 3 & 9 \\ 0 & 0 & 1 & 4 & -1 \\ 0 & 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$

e. $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 7 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

f. $\begin{bmatrix} 0 & 1 & 4 & 0 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

<u>Example 3:</u> **Write the system of equations represented by the augmented matrix. Then use back substitution to solve.**

$$\left[\begin{array}{ccc:c} \mathbf{1} & \mathbf{2} & \mathbf{-2} & \mathbf{-1} \\ \mathbf{0} & \mathbf{1} & \mathbf{1} & \mathbf{9} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{-3} \end{array}\right]$$

<u>Gaussian Elimination with Back Substitution</u>

1. Write the augmented matrix of the system of equations.

2. Use elementary row operations to rewrite the augmented matrix in *row-echelon* form.

3. Write the system of equations that corresponds to the row-echelon form matrix, and use back substitution to solve.

<u>Example 4a:</u> Use Gaussian Elimination with back substitution to solve the system.

$$\begin{cases} 2x + 6y = 16 \\ 2x + 3y = 7 \end{cases}$$

Example 4b:

$$\begin{cases} 3x - 2y + z = 15 \\ -x + y + 2z = -10 \\ x - y - 4z = 14 \end{cases}$$

If you obtain a row with all zeros except the last entry, then the system has *no solution*.
It is also called *inconsistent*.

Example 5: The system corresponding to the following matrix is inconsistent. It has no solution.

$$\begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-5} & \mathbf{3} & \mathbf{9} \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{4} & \mathbf{-1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{6} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{1} \end{bmatrix}$$

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