Half-Angle Formulas

$$\sin\frac{u}{2} = \pm\sqrt{\frac{\mathbf{1-\cos u}}{\mathbf{2}}} \qquad \cos\frac{u}{2} = \pm\sqrt{\frac{\mathbf{1+\cos u}}{\mathbf{2}}} \qquad \tan\frac{u}{2} = \frac{1-\cos u}{\sin u} = \frac{\sin u}{1+\cos u}$$

The sign $(\pm)$ depends on the quadrant that $\frac{u}{2}$ lies.

Example 1: Determine whether the positive or negative square root should be selected.

a. $\sin 195° = \pm\sqrt{\frac{\mathbf{1-\cos 390°}}{\mathbf{2}}}$

b. $\cos(-10°) = \pm\sqrt{\frac{\mathbf{1+\cos(-20°)}}{\mathbf{2}}}$

Example 2: Find the exact value of the following:

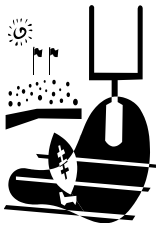
a. sin 195°

b. cos 105°

c. $\tan\frac{\pi}{6}$

Example 3: Find all solutions in the interval [0, 2π). $\cos^2 x = \sin^2 \frac{x}{2}$

Example 4: Find $\cos \frac{\theta}{2}$, given $\cos \theta = \frac{1}{4}$ and $\frac{3\pi}{2} \leq \theta \leq 2\pi$



Example 5: Ignoring air resistance, the range of a projectile fired at an angle θ with the horizontal and with an initial velocity of v_o feet per second is given by $\boldsymbol{r = \frac{1}{16} v_o^2 \sin\theta \cos\theta}$, where r is the horizontal distance (in feet) that the projectile will travel. A place kicker for the Stevenson Titan's football team can kick a football from ground level with an initial velocity of 78 feet per second. At what angle must the player kick the football so that it travels 188 feet?

Homework: pages 413-415 #53-71 odd, 77, 79