

## Fundamental Trigonometric Identities

### Cofunction Identities

$$\begin{array}{ll} \sin \theta = \cos\left(\frac{\pi}{2} - \theta\right) & \cos \theta = \sin\left(\frac{\pi}{2} - \theta\right) \\ \tan \theta = \cot\left(\frac{\pi}{2} - \theta\right) & \cot \theta = \tan\left(\frac{\pi}{2} - \theta\right) \\ \sec \theta = \csc\left(\frac{\pi}{2} - \theta\right) & \csc \theta = \sec\left(\frac{\pi}{2} - \theta\right) \end{array}$$

### Reciprocal Identities

$$\begin{array}{ll} \sin \theta = \frac{1}{\csc \theta} & \csc \theta = \frac{1}{\sin \theta} \\ \cos \theta = \frac{1}{\sec \theta} & \sec \theta = \frac{1}{\cos \theta} \\ \tan \theta = \frac{1}{\cot \theta} & \cot \theta = \frac{1}{\tan \theta} \end{array}$$

### Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta} \qquad \cot \theta = \frac{\cos \theta}{\sin \theta}$$

### Pythagorean Identities

$$\sin^2 \theta + \cos^2 \theta = 1 \qquad 1 + \tan^2 \theta = \sec^2 \theta \qquad 1 + \cot^2 \theta = \csc^2 \theta$$

### Even/Odd Identities

$$\begin{array}{lll} \sin(-\theta) = -\sin(\theta) & \cos(-\theta) = \cos(\theta) & \tan(-\theta) = -\tan(\theta) \\ \csc(-\theta) = -\csc(\theta) & \sec(-\theta) = \sec(\theta) & \cot(-\theta) = -\cot(\theta) \end{array}$$

Sine, cosecant, tangent and cotangent are odd functions. Cosine and secant are even functions.

**Example 1:** Use identities to find the exact value of the remaining trigonometric functions, if possible.

a.  $\sin x = \frac{1}{2}$  and  $\cos x > 0$

b.  $\csc \theta = -5$  and  $\cos \theta < 0$

We are now going to use identities to simplify trigonometric expressions.

**Tip:** Write trig functions in terms of sine and/or cosine before simplifying, when possible.

**Example 2:** Simplify  $\csc\theta\tan\theta$

**Example 3:** Simplify  $\csc t - \cos t \cot t$

**Example 4:** Simplify  $\frac{1-\sin^2 x}{\cot x}$

**Try This!** Simplify each of the following.

A)  $\cot x \sec x$

B)  $\sec x (\cos x - \cos^3 x)$

**Homework:** Page 377 #11-15 odd, 21, 37-52 all